Year 12 / IB1 intensive revision course – Mathematics HL/SL

This course is aimed at providing students with an opportunity to thoroughly revise their 1st–year IB topics during fully-customizable tutorial sessions, according to the request of students and/or parents. The course will allow students to "catch up and patch up" and address misunderstandings that they may have accrued during their first year of IB studies.

Students will be pre-evaluated based on their performance on a customized multiple-choice test, with a mix of adapted IB past exam questions and in-house questions. These questions are exclusively based on topics that students should have covered at school. Samples of the diagnostic tests can be found on our Resources page at www.akademiatuition.com.

A standard course will feature 8 to 12 hours of revisions on typical first year school topics, with emphases on the following points:

- **A deep understanding that the GDC and formula book do not help you RECOGNIZE how to proceed.**
- Techniques on solving HL problems, such as:
  - Mathematical induction problems involving divisibility and inequalities – unleashing your creativity in mathematics
  - Complex number problems – treating complex numbers as surds and as vectors, recognizing rotation and rotational symmetry (multiplication, power and root) and reflection symmetry (conjugates)
  - Functions and graphs – knowing the basic shapes and features of function graphs
  - Vector problems – visualizing 3D and understanding the importance of parameters during problem solving
  - Calculus – from basic algebra to setting up advanced differentiation and integration problems
- Techniques on solving standard problems for SL and HL topics, such as:
  - Converting a description into a math equation
  - Disguised quadratics, polynomials and simultaneous equations
  - Venn and tree diagrams – deciding which tool to choose

The following pages contain sample excerpts from our course material.

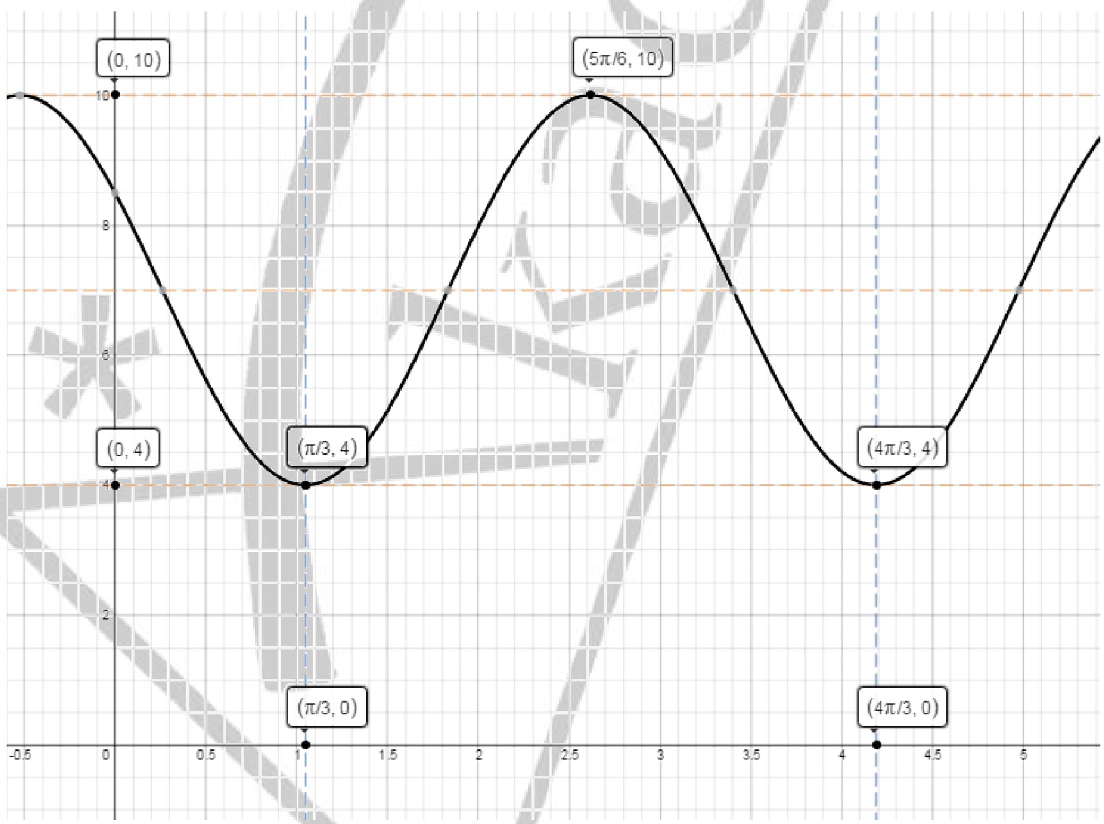
Excerpts from our course material

Example 1. Graph sketching: sketch the graph $y = -3\cos\left[2\left(x - \frac{\pi}{3}\right)\right] + 7$. [6]

Akademia experts' take: Before the cosine curve is sketched, it is highly advisable for students to recognize the framework of the graph – the framework of the graph is more important than the graph itself. Also, students need to recognize the meaning of each number / value / sign within the function expression. Specifically:

- A negative–cosine curve means that during the completion of 1 period, the graph starts from the lowest point, rises to the top after half a period, before sinks back to the lowest point;
- +7 means the vertical mid–point of the graph is at y = 7;
- (–)3 is the amplitude and means that vertically, the graph will have a span of 7 ± 3, which is from 4 to 10;
- 2 means that the period of the graph is $\frac{2\pi}{2} = \pi$;
- $\frac{\pi}{3}$ is a horizontal shift to the right, which means that the x–starting point (a low–point (trough)) should be placed at x = $\frac{\pi}{3}$. One period = π, which means that the graph will be at a low–point (y = 4) again when x = $\frac{4\pi}{3}$;

The finished product is shown below:



Example 2: If f(x) = ln(x + 6) and g(x) = x^2 – 2x, find the domain, range and inverse of f(g(x)). [6]

Akademia experts' take: This problem involves a composite function. Although f(x) and g(x) are both standard functions, it can still be tricky for students to recognize how to use the relevant features of each function to put together the big picture.

It should be obvious that f(g(x)) = ln(x^2 – 2x + 6).

For determining the domain, we need to be concerned about the possibility of applying logarithm to zero or a negative value. Upon further inspection, however, it should be clear that x^2 – 2x + 6 is always greater than 0, as the square–completed form of x^2 –2x + 6 is $(x – 1)^2$ + 5, which shows that the content of the natural log can only take on all values greater than or equal to 5. Therefore, we have no worries regarding the invalid application of logarithms to a negative number, and we can conclude that the domain of the composite f(g(x)) = ln(x^2 – 2x + 6) is "all real x".

For determining the range, we can find the inverse first and then analyze its domain. Alternatively, we can recognize that natural log, as a function, is strictly increasing, which means that the smallest input (5) will lead to the smallest value (ln 5) within the range. Hence the range of the inverse is y ≥ ln 5.

The determination of the inverse involves fairly standard algebra. Reversing the roles of x and y and rearranging for y as the subject:

$$x = ln(y^2 - 2y + 6)$$

$$e^x = y^2 - 2y + 6$$

To continue from here, we must recognize the need to complete the square for the right hand side:

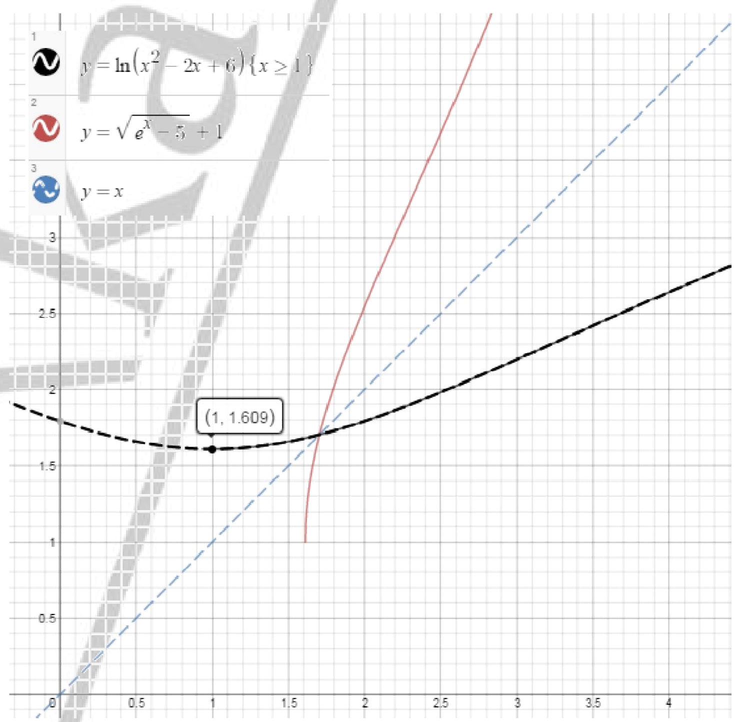
$$e^x = (y - 1)^2 + 5$$

$$\pm\sqrt{e^x - 5} = y - 1$$

For the final relation to be a function, we will need to discard one root (conventionally the negative root):

$$\sqrt{e^x - 5} = y - 1$$

y = $\sqrt{e^x - 5}$ + 1 as requested.



Example 3. A tangent to the curve $y = f(x) = x^3$ has a y–intercept $c = -2$. Find the equation of this tangent. [6]

Akademia experts' take: This example is a moderately challenging external–tangent problem. We do not know the gradient of the tangent – in fact that's what we need to find. So, let us let the equation of the tangent be $y = mx - 2$.

We do not know the contact point. However, even though the question did not ask for the point of contact, we need to recognize the possibility to express the coordinates of our point of contact as a single variable (as seen below). This way, we can set up simultaneous equations involving the gradient and this other variable.

Let the point of contact have an x–coordinate of k.

Since $f(x) = x^3$, the y–coordinate is k^3, and the point of contact is (k, k^3).

- In order for a straight line to be a tangent to a curve, the line and the curve need to make contact:
  The y–value of tangent at $x = k$ equals to the y–value of $f(x)$ at $x = k$:
  $mk - 2 = k^3$ (equation 1)
- In order for tangency to be achieved, the gradient of the tangent must equal to the derivative of the graph at $x = k$:
  $m = 3k^2$ (equation 2)

The remaining of the problem is fairly trivial – solving simultaneous equations involving substitution:

$(3k^2)k - 2 = k^3$

$2k^3 = 2$

$k = 1$

Hence the point of contact is (1, 1), the value of m is 3, and the equation of the straight line is $y = 3x - 2$.