Pre-IB Intensive HL/SL Mathematics Course

Our summer pre-IB course is designed to help students who have just completed their IGCSE or IB-MYP studies, and are preparing for the big jump to IB-level studies where the scope of knowledge required is broader and deeper, leading to much more challenging exams.

With our natural talents, passion in mathematics, effective communication skills and vast teaching experience, Akademia experts have been able to strengthen the mathematical ability of students. Our success stories range from novice students making great leaps in their progresses, to above average students getting another boost and vaulted to the very top of their year group.

Being well aware of the great leap between IGCSE–level mathematics and IB mathematics, Akademia experts have developed a summer series of mathematics, involving rigorous training of algebra, graphing and interpretation skills – three indispensable aspects of every single math exams at the IB level. We want our students to build on their IGCSE–level knowledge and develop a firm grasp of basic mathematical principles, such as equation solving, functions, graph sketching, and real–life application of mathematics. Our ultimate goal is to well–prepare students for **independent multiple–step problem solving**, a skill that will prove highly useful throughout a student's IB diploma studies.

Students and parents will be given **free** consultations, over the phone or in person, in order for us to identify the course participants' background, strengths and weaknesses. A 12-hour course will then be tailored by our course experts, based on the students' IB subject selection, aspirations, topics of interest, and our own recommendations.

The course material will be delivered at a level and pace that is suitable for all attendees. Capable students who run ahead will be given extra challenges in order to keep them at the top of the pack.

Course contents

A) Basic algebra

- Simultaneous equations and polynomial equation solving
- Algebra skills: index, exponents, logarithms, surds, factorizing and expanding, algebraic fractions
- Sequences, series, binomial expansions
- (HL only) Counting principles, introductory complex numbers, mathematical induction

B) Functions and graphs

- Domain, range, inverse, applications
- Sketching of basic mathematical models: polynomials, reciprocals, exponents and logs, trigonometric functions

C) Geometry and trigonometry

- Circle geometry – sectors and areas, radian measures
- Trigonometry – SOHCAHTOA, triangle solving, introduction to the unit circle
- Compound angle formulas and other problem solving

D) Introduction to calculus

- Meaning of differentiation and integration
- Basic algebraic manipulations: product rules, chain rules, quotient rules
- Applications: Rates of change, optimization problems, physical kinematic applications

Example of problem solving

1. (No calculator) Find the area of smallest triangle who has a 120-degree angle and whose sides are integers that are consecutive terms in an arithmetic sequence. [9]

Akademia's experts' take:

This is the type of question that fazes a lot of students. It combines ideas of sequences (3 sides with length values in arithmetic sequence), triangle solving (application of cosine rule and area rule) and some realization of properties of a triangle (the longest side is across the widest angle). The challenge here is for the student to not only properly set up a relevant equation, but also carefully carry out algebraic manipulation in solving for a meaningful solution.

The actual solution:

- Recognize the need to assign the sides to be something like a, a + d, and a + 2d (a – d, a and a + d also work) in order to fulfill the sequence properties.
- Realize that the longest side (a + 2d) is opposite to the 120° angle.
- Recognize the need to use cosine rule to relate a and d.
- Recognize that $\cos 120^{\circ} = -0.5$ since the problem is to be solved without a calculator.
- Use cosine rule to set up $(a + 2d)^2 = a^2 + (a + d)^2 - 2a(a + d)\cos 120^{\circ}$.
- Expand both sides of the equation above to obtain $a^2 + 4ad + 4d^2 = a^2 + a^2 + 2ad + d^2 + a^2 + ad$.
- Group like terms to obtain to $2a^2 - ad - 3d^2 = 0$.
- Factorize the left hand side to obtain $(2a - 3d)(a + d) = 0$.
- $a = 3d/2$ is the only reasonable solution.
- The smallest positive–integer solution set is d = 2 and a = 3.
- Hence the 3 sides have lengths 3, 5 and 7 respectively.
- Using the area formula A = 0.5absinC with a = 3, b = 5 and C = 120° gives area = $\frac{15\sqrt{3}}{4}$.

2. Consider the graph y = f(x) = $\frac{5x-15}{x-4}$.

a) Sketch f(x), clearly indicating all asymptotes and intercepts.

Consider the finite region between the x–axis, the y–axis and f(x).

b) Find the area of this region.

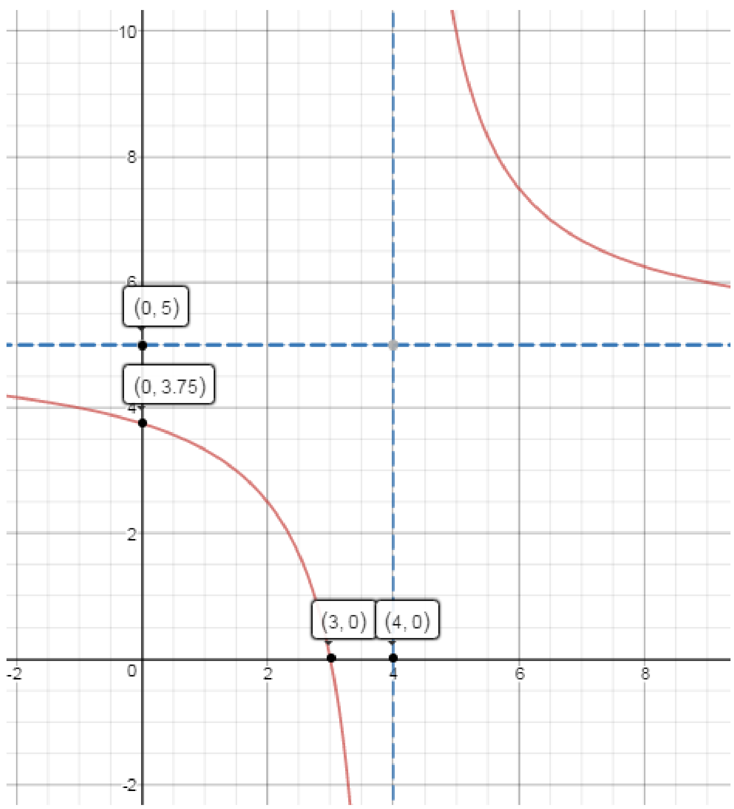
c) Consider rectangles OABC inscribed within this region, with A positioned on the x–axis, B being a point on f(x), and C located on the y–axis. Find the area of the biggest possible rectangle.

Akademia's experts' take:

The first two parts of this question can be solved using standard GDC techniques, while the last part of the question involves setting up and solving an optimization problem. Many will be unsure how to set up a meaningful expression that they can apply calculus skills and differentiate – an aspect of problem solving that typical students are weak at, and something that our course is going to emphasise.

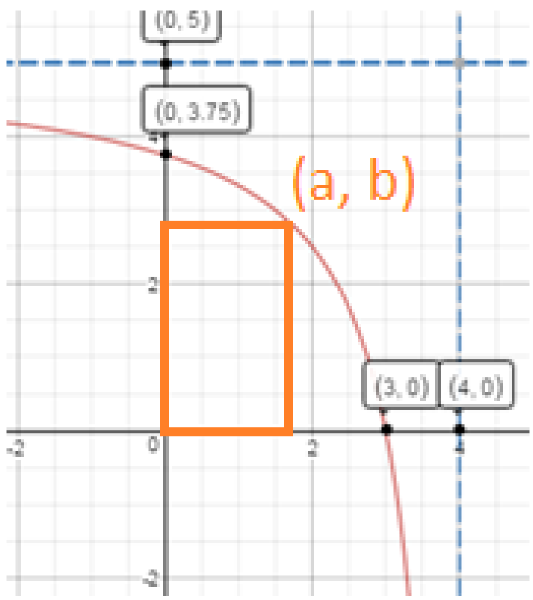
The actual solution:

a) This is a classical linear–over–linear reciprocal graph: asymptotes: x = 4, y = 5; Intercepts: y = 3.75, x = 3. While IB students will eventually have to produce this sketch without calculator aid, at this stage of learning, a GDC–based plot (based on CIE 0607 – International Mathematics techniques) will be acceptable.



b) This is an integration question for students with further mathematics (additional mathematics) background. Students can use the GDC to solve for an approximation trivially (area = 8.07 square units correct to 3 s. f.). If, instead, this question is to be solved without a calculator, students need to realize the need to use techniques such as long division to help rewrite $\frac{5x-15}{x-4}$ as $5+\frac{5}{x-4}$, which may be trivially integrated to give area = $[5x+5\,ln|x-4|]_0^3 = 15 \ - 5\ln 4$.

c) Most students just blindly differentiate f(x) simply because they see the word "maximum" within the question. Instead students must recognize the need to introduce a general point on the lower–left branch of the graph that defines the top right corner of the rectangle, before they can set up an area expression that can be meaningfully differentiated.



- Let B(a, b) be the position of the top–right corner of the rectangle. Since point B lies on the original function, we can replace its y–coordinate (b) with $\frac{5a-15}{a-4}$. Thus the coordinates of the point B can be rewritten as $(a, \frac{5a-15}{a-4})$.

- Students should be able to deduce the equation for the area of the rectangle: $A = a(\frac{5a-15}{a-4})$.

- Using quotient rule allows students to deduce that $\frac{dA}{da} = \frac{(a-4)(10a-15)-(5a^2-15a)(1)}{(a-4)^2}$.

- Solving $\frac{dA}{da} = 0$ results in $5(a^2 - 8a + 12) = 0$, yielding a = 2 or 6 (rejected as 6 is outside the domain of interest).

- A 1st derivative test shows that a = 2 indeed gives maximum area, which can be found to be 4.5 square units.