**Mathematics SL Diagnostic test – Non–Calculator paper (Paper 1)**

**Section A**

1. Find a value such that $\sum_{k=0}^{\infty} a^{2k+1}$ equals 1.2. [6]

2. Solve the simultaneous equations $\log_3 A + \log_9 B = 5$, $\log_9 A + \log_3 B = 5.5$ [6]

3. Consider the function $y = f(x) = \frac{2^{x+1}+8}{2^x+1}$.

a) State the domain of the function. [1]

b) State the value of f(0). [1]

c) Find an expression for the inverse of f(x). [6]

4. 3 boats A, B and C, depart a pier O (0, 0) at the same time. The velocity vectors of A, B and C are (i + 7j) m $s^{-1}$, (ai + bj) m $s^{-1}$ and (5i + 5j) m $s^{-1}$, respectively, with a and b being positive constants.

a) Show that boats A and C have the same speed. [2]

It's given that boat B also sails with the same speed, and that the path of B bisects the acute angle AOC.

b) Find the velocity vector of boat B. [5]

5. Find the point on the graph of $y = f(x) = \frac{1}{2\sqrt{x}}$ that's closest to the origin. [6]

6. Solve the equation $\tan^2(x - \frac{\pi}{4}) = \frac{1}{3}$, for $0 \le x \le 2\pi$. [5]

7. Find the finite area between the parabolas $y = x^2 + 7x$ and $y = -x^2 - 3x - 8$. [6]

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**Section B**

8. This question is about sketching a rational function graph.

a) Sketch the graph $y = f(x) = \frac{2x+3}{x+6}$, clearly indicating all asymptotes and intercepts. [5]

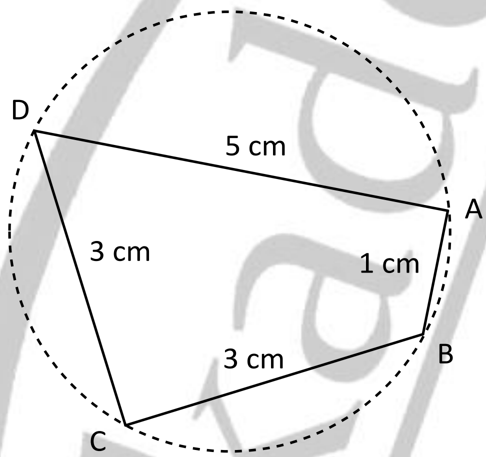
b) State the domain, range and inverse of y. [3]

c) Solve the equation $f^{-1}(x) = 7$. [2]

d) Find k such that the line $y = kx + 14$ is tangent to f(x). [6]

e) Find, if it exists, a line that goes through the origin and is tangent to f(x) at some point. [4]

9. It's given that ABCD is a cyclic quadrilateral inscribed on the circumference of a circle, with AB = 1 cm, BC = 3 cm, CD = 3 cm and DA = 5 cm. Given that the sum of opposite angles of cyclic quadrilaterals equals to $180^o$,......




a) Show that $\cos\angle ABC = -\frac{2}{3}$, and state the value of $\cos\angle ADC$. [7]

b) Calculate the area of ABCD, giving your answer in surd form. [4]

c) Calculate the radius of the circle, giving your answer in surd form. [4]

10. This question is about the differentiation of a polynomial.

a) Find the points on the graph $f(x) = x^4 - 4x^3$ where i) $f'(x) = 0$; ii) $f''(x) = 0$. [4]

b) Classify each point as a maximum, minimum, or a point of inflection, and justify your answers in each case. [6]

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**Mathematics SL Diagnostic test –Calculator paper (Paper 2)**

**Section A**

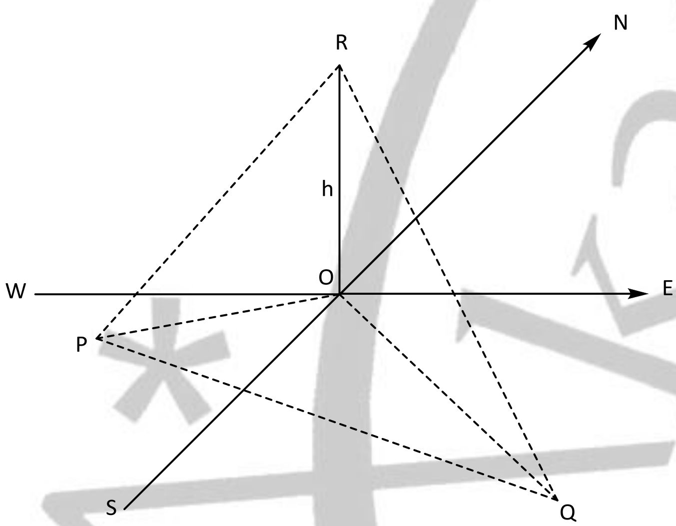
1. Find the values of n and k if $(x + 2)^n(x - k)$, when expanded, gives $-192 - 448x$ + higher powers of x. [6]

2. Find the sum of all positive integers, up to and including 240, that are divisible by either 4 or 6. [6]

3. A culture of bacteria increases its population by 8% every 14 minutes. Find, correct to the nearest minute, the time it takes for the bacteria population to triple. [4]

4. Consider the following diagram. A bird standing at the top of a wire pole OR, height = h, is being watched by two cats located at P and Q, respectively. The bearing of cat P is 240 (S $60^\circ$ W) and the bearing of cat Q is 130 (S $50^\circ$ E). The angle of elevation from P to R is $40^\circ$ and the angle of elevation from Q to R is $25^\circ$. The distance PQ is 20 metres.

Find the height, h, of the wire pole, and find the angle OPQ. [7]




5. This question is about integration of an unfamiliar function.

a) Differentiate $y = f(x)= xe^{-2x}$. [3]

b) Use your result in part a) to find the exact value of $\int_0^1 xe^{-2x}\,dx$. [5]

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6. In a population, 5% of the individuals are carriers of a genetic disease. A biochemical blood test is developed to test for carriers. Clinical trials show that the test is accurate 98% of the time if the individual carries the disease, and is accurate 96% of the time if the individual does not carry the disease.

a) What's the chance that a random person gets tested positive? [2]

b) An individual has now been tested positive for being a carrier the disease. What is the chance that he actually is a carrier? [3]

7. This question is about logarithmic functions.

a) Use the change of base formula to help find f'(x) given $f(x) = \log_2(x)$. [2]

b) Find the value of m such that $y = \log_m(x)$ is tangent to the graph of its own inverse $y = f^{-1}(x)$. [7]

**Section B**

8. At 12:00 a boat starts sailing from the origin (0, 0) with the vector equation B: r = t(i + j). Beacons M and N have position vectors 5i + 2j and 10i + j, respectively.

a) Draw a diagram with x and y axis drawn to scale to indicate the path of B, and the location of M and N. [2]

b) Find t such that the boat is the same distance from each beacon? [5]

c) Let y = cos(∠MBN). Show that $y = \frac{f(t)}{\sqrt{g(t) \times h(t)}}$ where f(t), g(t) and h(t) are quadratic expressions to be found. [6]

d) Sketch a graph from t = 0 to 10 that shows how y varies with time. [3]

e) Hence find the maximum and minimum values of ∠MBN. [4]

9. The east-west position of a toy car, in meters, is given by the equation $s(t) = -t^3 + 9t^2 - 24t + 20$. East is designated to be the positive direction in this question. An observing person is standing at s = 0.

a) Find v and a in terms of t. [3]

b) Find the 2 times when the toy car is momentarily at rest? [2]

c) Find when the toy car is moving with the most positive velocity? [3]

d) Find the change in position after 6 seconds. [1]

e) Find the total distance travelled after 6 seconds, and explain why this answer is not the same as the answer in part f. [3]

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10. Petra and Vince play a game as follows:

- Petra rolls two fair dice, notes the larger number showing, subtracts 1 from this number, and uses this result as her score.
- Vince rolls one die, and uses the number showing as his score.
- The person with the higher score wins the round. The round is a draw if the two scores are identical.
- When Petra wins, Vince has to pay her \$m. When Vince wins, Petra has to pay Vince \$5.40. When the score is a tie, no one has to pay anything.

a) Calculate the probability that Petra wins a particular round. [5]

b) Find the value of m such that the game above is fair to both. [3]

c) The two now play two consecutive rounds. Given that the results of the two rounds are different, find the probability that each person won one round. [5]

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